"An Account of the Base Observations made at the Kew Observatory with the Pendulums to be used in the Indian Trigonometrical Survey." By BALFOUR STEWART, M.A., LL.D., F.R.S., Superintendent of the Kew Observatory, and BENJAMIN LOEWY, Esq. Received June 13, read June 15, 1865.

Her Majesty's Indian Government, on the recommendation of the Royal Society, lately decided that pendulum observations shall be made at different stations in India in connexion with the Great Trigonometrical Survey of that country.

The object of these proposed observations may be stated in a very few words. The labours of those engaged in the Trigonometrical Survey have already disclosed the fact that the direction of the plumb-line in the northern stations of India was influenced to some extent by the mass of the Himalayas, and it was therefore thought highly desirable that the influence of these mountains upon the *intensity* of terrestrial gravity should be investigated in addition to their influence upon its *direction*. The propriety of this view will at once be evident, if we reflect that by knowing the change produced not only upon the direction of gravity, but also on its intensity, we know at once all the particulars of the disturbing mountain-force both as regards magnitude and direction.

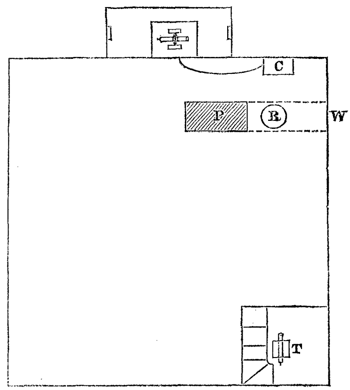
It was therefore with the view of ascertaining the alteration which these mountains might cause upon the intensity of gravity that the Indian pendulum observations were decided upon. In consequence of this decision, Captain Basevi, R.E., and first assistant in the Survey department, was appointed to superintend the observations, and instructed to repair to the Kew Observatory previously to his departure for India, in order to become acquainted with the necessary instruments, their adjustment, and the method of observing with them.

After attending daily at the Observatory from the beginning of September to the middle of November, this officer was perfectly instructed in every particular necessary for the practical part of these observations, as well as for their calculation and reduction. He was then obliged to leave for India, being prevented by his early departure from making the necessary base determinations with the instruments at Kew Observatory.

The best arrangement of apparatus formed the subject of careful discussion with Colonel Walker, Superintendent of the Indian Survey; and the experimental arrangements ultimately adopted received the sanction of this officer, who, besides suggesting several improvements, made himself thoroughly acquainted with all the details of the apparatus.

A room suitable for these observations was constructed in the south-east corner of the Observatory, the expenses of which were defrayed from the Government Grant Fund of the Royal Society.

The following simple diagram will be sufficient to show the experimental arrangement in the Pendulum-room.



C is the place for the clock, which is connected with the transit-instrument; P is the pillar bearing a slab attached also to the wall at W, to which the receiver (R) is rigidly fixed.

T is the telescope for the observations of the coincidences, mounted on a pillar which stands in a depression, so that the observer is not under the necessity of kneeling down during the observation.

In every other part the arrangement is entirely similar to that described by General Sabine in the Philosophical Transactions for 1829,—with this difference, that the receiver was in our experiments a copper one with glass windows. The whole of the apparatus was made by Mr. P. Adie, who deserves the highest praise for the excellent manner in which the work was executed by him.

The pendulums used were those marked No. 1821 and No. 4, used formerly by General Sabine in different parts of the globe. The former was also used by Mr. Airy in his Harton experiments.

Method of registering and reducing the Observations.

The manner in which the number of vibrations, made by a detached pendulum, are determined from a series of observed coincidences with the pendulum of a clock has been so often described, that we may refer to the writings of Kater, Sabine, Baily, and others on the subject. The established methods have been followed throughout in these experiments, and

the only change introduced was a very slight one, with the view of obtaining a more correct arc of vibration.

It is usual to observe the arc a little after the coincidence, which does not give the true arc corresponding to it. To obviate this, the arc was read in our series about 30 seconds before, and again 30 seconds after each observed coincidence, marking first the right edge of the tailpiece and then the left one. If we call these four readings of the scale a, b, c, d, we may consider

$$\frac{(a \sim b)+(c \sim d)}{2}$$

as a very exact representation of the reading for the *arc* at the instant of the coincidence.

The adjustment of the diaphragm, disk, and tailpiece was made very carefully at the commencement of the experiments. Nevertheless it was found slightly deranged when the pendulum was reversed. In this case, as is well known, the disappearance and reappearance of the disk are not each instantaneous; but we see first one side of the disk, then the other disappear, and in the same order reappear, so that we have four events, of which, calling the time of their happening respectively $\alpha, \beta, \gamma, \delta$, either

$$(1)\ \frac{\alpha+\beta+\gamma+\delta}{4}, \quad \text{or } (2)\ \frac{\alpha+\delta}{2}, \quad \text{or } (3), \text{ lastly, } \frac{\beta+\gamma}{2}$$

will give us the *time* of coincidence. In a few sets of our series the first formula was used; but it was soon found that the correct registration in such a case is a matter of the greatest difficulty, and it was therefore thought in one instance preferable to stop the clock and repeat the adjustment, and afterwards a similar derangement was rectified by a lateral motion of the observing telescope. With a few trials, using a few successive coincidences for the purpose, it is quite possible to adjust the whole to the greatest nicety without stopping the clock.

The reduction of the observations was made entirely after the manner of former experimenters. It comprises the following corrections:—

A. *Correction of the observed arc-readings and reduction of the vibrations to infinitely small arcs.* In the first place, the scale for reading the arc being behind the tailpiece of the pendulum, the registered readings are too large.

Let D be the distance of the scale from the object glass of the telescope, d its distance from the tail of the pendulum, O the observed reading of the *whole* arc on the scale graduated from end to end, S the distance of the indicating-point of the tailpiece from the knife-edge, then the true arc, or more correctly *semiarc* observed $(=\alpha)$, through which the pendulum moved from the vertical, is given by the formula

$$\text{tang}\,\alpha=\frac{\text{O}(\text{D}-d)}{2\text{DS}},$$

expressing all distances in inches, into which the scale was divided.

The values of $\frac{D-d}{2DS}$ were determined for each pendulum from accurate measurements, and are

$$\text{For pendulum No. 4 in position, face on} = \frac{98{\cdot}74}{2\times 100{\cdot}22\times 49{\cdot}89}$$

$$\text{,, ,, ,, ,, face off} = \frac{99{\cdot}67}{2\times 100{\cdot}22\times 49{\cdot}89}$$

$$\text{,, ,, No. 1821 ,, face on} = \frac{99{\cdot}47}{2\times 100{\cdot}22\times 49{\cdot}3}$$

$$\text{,, ,, ,, ,, face off} = \frac{98{\cdot}95}{2\times 100{\cdot}22\times 49{\cdot}3}.$$

The logarithms of these expressions were added to those of the observed readings for the logarithm of the tangent of α.

In the next place, the reduction to infinitely small arcs was deduced from the well-known formula,

$$\text{Number of infinitely small vibrations} = n + n\times\frac{M \sin(\alpha+\alpha')\sin(\alpha-\alpha')}{32(\log\sin\alpha - \log\sin\alpha')}\,*,$$

where M denotes the logarithmic modulus $=0{\cdot}4342945$; α the initial, and α' the final semiarc of vibration, expressed in degrees, minutes, and seconds, n being the number of observed vibrations ; and to obtain a more correct result from this formula, the calculation was made *for each interval* between two successive observations.

B. The *rate of the clock* was determined from a series of observations of star-transits, the results of which are given in Table I. The rate was somewhat unequal during the experiments, the range being equal to $\frac{6}{10}$ths of a second ; and besides, the unfavourable state of the weather occasioned longer intervals between the observations than was desirable. To free the results as far as possible from any errors arising from this source, the rates were represented in a series, as shown in Table II., which also gives the actual number of vibrations made by the sidereal clock in a mean solar day, as deduced from the following formula :—

$$\text{Number of vibrations in a mean solar day} = N' = 86636{\cdot}5554\left(1-\frac{r}{86400}\right),$$

where r is the observed rate, which in our case was a losing one throughout the whole of the observations.

If we now call V' the number of observed vibrations of the clock-pendulum from beginning to the end of one experiment, V the number of observed vibrations of the detached pendulum during the same time, corrected for the amplitude of the arc, and finally N' the number of actual vibrations of the clock in a mean solar day at the date of the experiment, found as above, we have for the number of infinitely small vibrations of

* See Memoirs of the Royal Astronomical Society, vol. vii. p. 22.

the detached pendulum during a mean solar day the following proportion:

$$V' : V :: N' : N,$$

$$N = \frac{VN'}{V'}.$$

TABLE I.—List of Transits observed in connexion with the Pendulum Experiments for India, and clock-rates deduced from them.

Date.	Name of Star.	Right Ascension.	Time of passing mean wire.	Sum of instrumental errors.	Error of clock.	Mean rate deduced.
1865.		h m s	h m s	s	m s	
January 7.	β Arietis	1 47 12·40	1 47 7·96	+0·76	− 3·68	
	α Arietis	1 59 35·42	1 59 30·96	+0·70	− 3·76	
	ξ^2 Ceti	2 20 60·28	2 20 55·42	+1·03	− 3·83	
	α Persei	3 14 44·47	3 14 40·84	−0·17	− 3·80	
	η Tauri	3 39 29·68	3 39 25·16	+0·68	− 3·84	
	γ′ Eridani	3 51 45·34	3 51 40·06	+1·48	− 3·82	
	α Tauri	4 28 12·58	4 29 39·4	+0·85	− 3·85	
„ 8.	η Bootis	13 48 15·68	13 48 9·98	+1·06	− 4·64	
	α Bootis	14 9 30·38	14 9 24·72	+1·04	− 4·62	
	ρ Bootis	14 26 0·57	14 25 55·16	+0·81	− 4·60	s
	ε Bootis	14 39 5·26	14 38 59·68	+0·89	− 4·69	−1·81
„ 9.	η Bootis	13 48 15·71	13 48 8·4	+0·84	− 6·47	
	α Bootis	14 9 30·42	14 9 22·98	+0·82	− 6·62	
	ρ Bootis	14 26 0·61	14 24 53·74	+0·55	− 6·32	
	ε Bootis	14 39 5·30	14 38 58·2	+0·64	− 6·46	−1·83
„ 13.	α Bootis	14 9 30·54	14 9 16·1	+0·97	− 13·47	
	ε Bootis	14 39 5·43	14 38 51·02	+0·81	− 13·60	−1·76
„ 14.	ε Bootis	14 39 5·46	14 37 4·56	+0·88	−2 0·02	
	α^2 Libræ	14 43 25·37	14 41 23·70	+1·65	−2 0·02	
	υ Piscium	1 34 25·30	1 32 23·44	+1·12	−2 0·74	
	β Arietis	1 47 12·31	1 45 10·66	+0·79	−2 0·86	
	α Arietis	1 59 35·33	1 57 33·8	+0·73	−2 0·80	
	ξ^2 Ceti	2 21 0·20	2 18 58·28	+1·06	−2 0·86	
	γ Tauri	3 39 29·61	3 37 27·98	+0·71	−2 0·92	−1·64
„ 16.	ρ Bootis	14 26 0·85	14 23 56·72	+0·79	−2 3·34	
	ε Bootis	14 39 5·52	14 37 1·16	+0·87	−2 3·49	
	α^2 Libræ	14 43 25·43	14 41 20·18	+1·65	−2 3·60	−1·76
„ 17.	α Bootis	14 9 30·67	14 9 45·86	+1·13	+ 16·32	
	ρ Bootis	14 26 0·89	14 26 16·3	+0·92	+ 16·33	
	ε Bootis	14 39 5·55	14 39 21·0	+0·98	+ 16·43	
	α^2 Libræ	14 43 25·46	14 43 40·22	+1·69	+ 16·45	
„ 19.	η Tauri	3 39 29·55	3 39 41·06	+0·57	+ 12·08	
	ε Tauri	4 20 46·05	4 20 57·54	+0·69	+ 12·18	
	α Tauri	4 28 12·49	4 28 23·78	+0·76	+ 12·05	−1·67
„ 20.	α Persei	3 14 44·22	3 14 54·92	−0·33	+ 10·39	
	γ′ Eridani	3 61 45·20	3 51 54·16	+1·41	+ 10·37	
	ε Tauri	4 20 46·04	4 20 55·6	+0·81	+ 10·37	
	α Tauri	4 28 12·48	4 28 21·94	+0·83	+ 10·29	
	α Aurigæ	5 6 46·01	5 6 56·2	−0·03	+ 10·16	−1·77
„ 22.	α Persei	3 14 44·18	3 14 51·28	−0·17	+ 6·93	
	η Tauri	3 39 29·52	3 39 35·74	+0·67	+ 6·89	
	γ′ Eridani	3 51 45·18	3 51 50·66	+1·46	+ 6·94	
	ε Tauri	4 20 46·02	4 20 52·16	+0·88	+ 7·02	
	α Tauri	4 28 12·46	4 28 18·38	+0·94	+ 6·86	−1·71

TABLE I. (*continued.*)

Date.	Name of Star.	Right Ascension.	Time of passing mean wire.	Sum of instrumental errors.	Error of clock.	Mean rate deduced.
1865.		h m s	h m s	s	m s	
January 28.	β Tauri	5 17 47·76	5 17 46·36	+0·96	− 0·44	
	δ Orionis	5 25 8·50	5 25 6·5	+1·42	− 0·58	
	ε Orionis	5 29 23·66	5 29 21·72	+1·44	− 0·50	s
	α Orionis	5 47 53·80	5 47 51·96	+1·31	− 0·53	−1·22
Febr. 9.	γ' Eridani	3 51 44·90	3 51 29·06	+1·53	−14·31	
	ε Tauri	4 20 45·78	4 20 30·58	+0·81	−14·39	
	α Tauri	4 28 12·23	4 27 56·98	+0·87	−14·38	
	ι Aurigæ	4 48 14·34	4 47 59·3	+0·63	−14·41	
	ε Leporis	4 59 46·14	4 59 30·06	+1·79	−14·29	
	α Aurigæ	5 6 45·68	5 6 31·16	+0·22	−14·30	
	β Tauri	5 17 47·61	5 17 32·26	+0·88	−14·47	
	α Leporis	5 26 48·22	5 26 31·96	+1·73	−14·53	−1·16
„ 17.	ι Aurigæ	4 48 14·29	4 47 48·36	+0·41	−25·52	
	ε Leporis	4 59 45·99	4 59 18·68	+1·73	−25·58	
	α Aurigæ	5 6 45·52	5 6 19·98	−0·06	−25·60	
	β Tauri	5 17 47·47	5 17 21·36	+0·54	−25·57	
	δ Orionis	5 25 8·24	5 24 41·4	+1·22	−25·62	−1·40
„ 19.	μ Geminorum ..	6 14 49·62	6 14 20·16	+0·50	−28·98	
	γ Geminorum ..	6 29 56·79	6 29 27·08	+0·66	−29·05	
	α Canis Majoris	6 39 13·88	6 38 43·26	+1·50	−29·12	−1·69
„ 20.	ι Aurigæ	4 48 14·13	4 47 42·58	+0·82	−30·73	
	ε Leporis	4 59 45·93	4 59 13·4	+1·84	−30·69	
	α Aurigæ	5 6 45·43	5 6 14·26	+0·45	−30·72	
	β Tauri	5 17 47·42	5 17 15·7	+0·92	−20·80	
	α Leporis	5 26 48·03	5 26 15·48	+1·75	−30·80	−1·80

TABLE II.—Showing the rate of the clock, and the number of its vibrations during a mean solar day.

No. f exp.	Rate (sid. time).	No. of vibr. in a mean solar day =N'.	No. of exp.	Rate (sid. time).	No. of vibr. in a mean solar day =N'.	No. of exp.	Rate (sid. time).	No. of vibr. in a mean solar day =N'.
	s			s			s	
1	−1·76	86634·795	21	−1·16	86635·395	41	−1·40	86635·155
2	−1·76	86634·795	22	−1·16	86635·395	42	−1·40	86635·155
3	−1·76	86634·795	23	−1·16	86635·395	43	−1·46	86635·095
4	−1·76	86634·795	24	−1·16	86635·395	44	−1·52	86635·035
5	−1·64	86634·915	25	−1·16	86635·395	45	−1·52	86635·035
6	−1·64	86634·915	26	−1·16	86635·395	46	−1·58	86634·975
7	−1·64	86634·915	27	−1·16	86635·395	47	−1·58	86634·975
8	−1·76	86634·795	28	−1·16	86635·395	48	−1·64	86634·915
9	−1·76	86634·795	29	−1·16	86635·395	49	−1·64	86634·915
10	−1·67	86634·885	30	−1·16	86635·395	50	−1·64	86634·915
11	−1·67	86634·885	31	−1·16	86635·395	51	−1·64	86634·915
12	−1·77	86634·785	32	−1·16	86635·395	52	−1·69	86634·865
13	−1·73	86634·825	33	−1·16	86635·395	53	−1·69	86634·865
14	−1·69	86634·865	34	−1·16	86635·395	54	−1·69	86634·865
15	−1·26	86635·295	35	−1·22	86635·335	55	−1·69	86634·865
16	−1·24	86635·315	36	−1·22	86635·335	56	−1·69	86634·865
17	−1·22	86635·335	37	−1·28	86635·275	57	−1·75	86634·805
18	−1·20	86635·355	38	−1·28	86635·275	58	−1·75	86634·805
19	−1·18	86635·375	39	−1·28	86635·275	59	−1·75	86634·805
20	−1·18	86635·375	40	−1·34	86635·215	60	−1·75	86634·805

C. *Correction for temperature.*—Two thermometers were fixed, one to the lower, the other to the upper part of a brass bar, which was made by Mr. Adie, of precisely the same form as the pendulums.

The brass bar being fixed near the middle of the receiver, close to the swinging pendulum, every change in the temperature of the latter was of course shared by the brass bar, and indicated by the two thermometers, which were extremely sensitive and read to ·05 of a degree. The readings of these two thermometers were in the first instance corrected for index-error. The instruments having been very carefully compared with the Kew Standard, a table of index-errors was made from these comparisons, and, by interpolation, giving the errors from degree to degree. Another correction was applied for the observations in the exhausted receiver on account of the effect of exhaustion on the glass tubes of the thermometers. This effect was determined very accurately by a series of experiments at Kew, and found to be equal for both thermometers, and amounting to 0°·43 for a decrease in pressure of 29·210 inches. This correction is smaller than that assumed by General Sabine and the late Mr. Baily, who make it $\frac{3}{4}$ of a degree for the thermometers which they employed.

Our experiments showed the remarkable fact that the correction is by no means proportional to the decrease in pressure. The diminution of the pressure from 30·080 inches to 13·610, that is, by an amount of 16·470 inches, gave a correction of only 0°·052, while a further decrease of 12·820, bringing the pressure to 0·790 inch, gave for one thermometer 0°·377, and for the other 0°·385.

The mean of the upper and lower thermometer reading will give the temperature of the pendulum at the moment of the observations; and if we call t, t', t'', t''' the temperatures found in this manner for the successive observations, we have

$$\frac{t+t'}{2},\ \frac{t'+t''}{2},\ \frac{t''+t'''}{2}$$

as the most probable temperature during the interval between two consecutive observations. These intervals being of unequal length, we will call n, n', n'', n''', the number of coincidence-intervals which they contain; and calling t° the mean temperature of the whole experiment, we have

$$^{\circ} = \frac{n\left(\frac{t+t'}{2}\right)+n'\left(\frac{t'+t''}{2}\right)+n''\left(\frac{t''+t'''}{2}\right)\ldots\ldots}{n+n'+n''\ldots\ldots}$$

Table III. gives the mean temperature found in this manner for each experiment, and shows the mean of all observed temperatures for each pendulum, to which temperature all the experiments made with that pendulum have been reduced. For this reduction it would have been best if we had had an opportunity of swinging the pendulums at extremes of temperature, say about 50° distant from each other. But the desirability of sending the apparatus to India under the care of Mr. Hennessey, who left by the March mail,

prevented such a course, and we availed ourselves of the elaborate series of experiments on the temperature corrections of pendulums, made by General Sabine (*vide* Phil. Trans. 1830, p. 251), which gives 0·44 vibration per diem for each degree of Fahrenheit's scale. General Sabine found in a former series this correction nearer to 0·43; and he says, in the above mentioned publication, "The probable error which may be incurred by employing the correction 0·44 for each degree as now determined, can only be very inconsiderable; but when the differences of temperature amount to 50°, which is a case of actual experience in pendulum observations, the question of whether 0·43 or 0·44, for example, be the more correct value, involves an uncertainty in the ultimate result of no less than half a vibration a day."

The pendulums which we used were not those employed by General Sabine in his determinations, but they were made by the same maker at the same time, and very probably from the same kind of brass, and there cannot be the least doubt that the true correction will lie between 0·43 and 0·44. We have therefore adopted 0·435 for our reductions; and as the greatest difference in temperature between a single experiment and the mean is less than 11°, the greatest error would in this case amount only to $\frac{5}{100}$ths of a vibration per diem, an error too small to affect seriously the mean result of the whole.

At the same time we must state that, as Colonel Walker and Captain Basevi inform us, experiments will be made in India with both pendulums, to ascertain their exact constants with regard to expansion, and that our results will of course have then to be modified accordingly.

TABLE III.—Showing the Mean Temperature for each experiment, and the Mean of the whole series for each Pendulum.

Pendulum No. 1821.		Pendulum No. 4.	
No. of experiment.	Mean temperature.	No. of experiment.	Mean temperature.
1	57°·963	1	55°·869
2	54·573	2	52·692
3	53·750	3	52·116
4	54·460	4	53·586
5	52·890	5	54·269
6	52·300	6	51·047
7	52·233	7	51·432
8	53·339	8	52·093
9	53·397	9	53·972
10	49·154	10	50·794
11	49·075	11	45·631
12	51·649	12	50·120
13	49·789	13	50·455
14	50·795	14	57·081
15	47·723	15	56·737
16	48·813	16	58·894
17	48·398	17	59·412
18	48·843	18	60·954
19	48·203	19	60·597
20	46·264	20	62·560
21	46·515	21	65·551
22	50·892	22	59·162
23	50·077	23	61·489
24	50·352	24	62·183
25	54·051	25	64·151
26	55·714	26	66·690
27	55·536		
28	56·561		
29	58·344		
Mean	51·781	Mean	56·520

D. *Correction for pressure of air.*—This correction, as shown in the Phil. Trans. for 1832, is thus determined :—

Let β' denote the reading of the gauge for the mean of the experiments made in air, and β'' the same reading for the mean of the vacuum experiments; also let t° denote the mean temperature of all the experiments, both in air and vacuo, then the expression

$$\frac{\beta'-\beta''}{1+\cdot 0023(t^\circ-32)}$$

will denote very nearly the mean difference of density between the two sets of experiments.

Now if N′ denote the mean number of vibrations *in air* during a mean solar day, and N″ the mean number of vibrations *in vacuo* during the same time, then the constant for one inch of reduced pressure will be

$$C'=\frac{N''-N'}{\beta'-\beta''}\left(1+\cdot 0023(t^\circ-32)\right).$$

Hence if β denote the actual mean pressure for a single experiment and

t the mean temperature of that particular experiment, the final correction for that experiment will then be found

$$C = C' \times \frac{\beta}{1 + \cdot 0023(t-32)}.$$

The following Table (IV.) gives the elements for obtaining the constant C′ for both pendulums.

TABLE IV.—Elements for deducing the Constant C′ from the Experiments in Air and in the Exhausted Receiver.

Pendulum No. 1821, Position "Face on."							
In Air.				In the Exhausted Receiver.			
No. of experiment.	Number of corrected vibrations.	Pressure.	Temperature.	No. of experiment.	Number of corrected vibrations.	Pressure.	Temperature.
		in.	°			in.	°
I.	86063·849	29·793	57·96	I.	86072·053	2·165	49·15
II.	86063·881	29·381	54·57	II.	86072·725	1·325	49·07
III.	86063·818	29·193	53·75	III.	86072·643	1·402	51·65
IV.	86064·126	29·153	54·46	IV.	86072·917	0·943	49·79
V.	86063·771	29·048	54·05	V.	86072·846	1·944	50·80
VI.	86064·240	29·103	55·71	VI.	86072·495	0·905	47·72
VII.	86064·202	29·222	55·54				
VIII.	86064·138	29·271	56·56				
IX.	86064·015	29·362	58·34				
Means...	86064·015	29·281	55·66	Means...	86072·613	1·447	49·70

Pendulum No. 1821, Position "Face off."							
		in.	°			in.	°
I.	86064·362	28·906	52·89	I.	86073·049	0·781	48·81
II.	86064·206	28·718	52·30	II.	86072·769	1·461	48·40
III.	86064·354	28·990	52·23	III.	86072·349	1·031	48·84
IV.	86064·578	29·180	53·34	IV.	86072·385	0·814	48·20
V.	86064·423	29·068	53·40	V.	86072·200	0·960	46·26
VI.	86063·433	29·378	50·89	VI.	86072·289	1·513	46·51
VII.	86063·375	29·279	50·08				
VIII.	86062·738	29·005	50·35				
Means...	86063·934	29·065	51·94	Means...	86072·507	1·093	47·84

Pendulum No. 4, Position "Face on."							
		in.	°			in.	°
I.	86162·815	29·926	55·87	I.	86171·544	1·649	57·08
II.	86162·495	30·159	52·69	II.	86171·213	1·831	56·74
III.	86162·518	30·427	52·12	III.	86170·853	2·865	58·89
IV.	86162·351	30·459	53·59	IV.	86170·518	3·340	59·41
V.	86162·394	30·532	54·27	V.	86170·909	3·839	60·95
VI.	86162·774	29·718	60·60				
VII.	86163·228	29·709	62·56				
VIII.	86163·337	29·683	65·55				
Means...	86162·739	30·077	57·16	Means...	86171·007	2·705	58·61

TABLE IV. (*continued.*)

Pendulum No. 4, Position "Face off."							
In Air.				In the Exhausted Receiver.			
No. of experiment.	Number of corrected vibrations.	Pressure.	Temperature.	No. of experiment.	Number of corrected vibrations.	Pressure.	Temperature.
		in.	°			in.	°
I.	86162·460	30·631	51·05	I.	86172·510	1·491	50·79
II.	86162·187	30·638	51·43	II.	86171·026	1·973	47·75
III.	86162·994	30·640	52·09	III.	86172·383	0·575	45·63
IV.	86162·933	29·355	59·16	IV.	86172·236	0·637	50·12
V.	86163·613	29·303	61·49	V.	86171·482	0·535	50·45
VI.	86163·484	29·306	62·18				
VII.	86163·480	29·477	64·15				
VIII.	86163·757	29·647	66·69				
Means...	86163·113	29·874	58·53	Means...	86171·927	1·042	48·95

Determination of the Constant C′ from the above Mean Results.

	N″−N′.	β′−β″.	1+·0023(t°−32).	Value of C′.
Pendulum No. 1821, Face on...	8·609	27·834	1·047564	0·324010
,, ,, ,, Face off...	8·573	27·972	1·041147	0·319096
Pendulum No. 4, Face on	8·268	27·372	1·059524	0·320040
,, ,, ,, Face off	8·814	28·832	1·050002	0·320988

E. The *reduction of the resulting number of vibrations to the sea-level* is calculated from

$$\frac{N}{R}\times h\times x,$$

where R is the earth's radius at the latitude of the Kew Observatory, h the height of the receiver above the mean level of the sea, and x a quantity which, with Dr. Young, may be assumed for a tract of level country to be $=\cdot666$ (*vide* Phil. Trans. for 1819, page 98).

This correction has been only applied to the ultimate mean number of vibrations of each pendulum.

Taking Bessel's value for the semiaxis major and the eccentricity of the earth, and $h=17\cdot5$ feet as given by measurement and the known height of our standard barometer, the logarithm of the factor for this correction is $\bar{7}\cdot7467623$.

Result.

Adopting the values for the reduction to a vacuum as found in Table IV., and applying the correction to those experiments, which were made in a highly rarefied medium, we find the following numbers of vibrations made by each pendulum in both positions in a *mean solar* day *in vacuo*, viz. for

Pendulum No. 1821, Face on, Exp.	I.	86072·728	Mean: 86073·064
	II.	86073·138	
	III.	86073·078	
	IV.	86073·211	
	V.	86073·450	
	VI.	86072·777	
„ „ Face off, Exp.	I.	86073·289	Mean: 86072·844
	II.	86073·218	
	III.	86072·668	
	IV.	86072·635	
	V.	86072·497	
	VI.	86072·756	
Pendulum No. 4, Face on, Exp.	I.	86172·043	Mean: 86171·822
	II.	86171·767	
	III.	86171·717	
	IV.	86171·524	
	V.	86072·061	
„ „ Face off, Exp.	I.	86172·969	Mean: 86172·249
	II.	86171·637	
	III.	86172·562	
	IV.	86172·432	
	V.	86171·647	

And reducing the means to the sea-level, we obtain the following

Final Result:—

Pendulum No. 1821,	Face on, 86073·112	vibrations	in a	mean solar day.
„ „ „	Face off, 86072·892	„	„	„
Pendulum No. 4,	Face on, 86171·870	„	„	„
„ „ „	Face off, 86172·297	„	„	„

Finally, we give an example of one experiment, with the mode of its reduction.

EXAMPLE OF THE OBSERVATIONS AND REDUCTIONS.

Experiment No. 12. January 20th, 1865. Pendulum No. 1821, Face on. In the exhausted Receiver. The clock making 86634·785 vibrations in a mean solar day.

Observation.

Number of coincidence.	Time of first disappearance.	Second disappearance.	Time of first reappearance.	Second reappearance.	Time of coincidence.	Reading of scale before		$a \sim b$.	Reading of scale after		$c \sim d$.	Thermometers.		Pressure.
						a.	*b.*		*c.*	*d.*		Upper.	Lower.	
	h m s	s	h m s	s	h m s							°	°	in.
1	2 6 36·5	36·5	2 6 58·5	58·5	2 6 57·5	0·60	2·86	2·26	1·29	3·53	2·24	53·15	53·05	0·768
2	12 3·5	3·5	12 5·5	5·5	12 4·5	0·62	2·84	2·22	1·30	3·53	2·23	53·20	53·10	
3	17 11·5	11·5	17 13·5	13·5	17 12·5	0·62	2·84	2·22	1·32	3·52	2·20	53·30	53·20	0·775
17	3 28 45·5	45·5	3 28 47·5	47·5	3 28 46·5	0·74	2·72	1·98	1·44	3·40	1·96	53·20	52·95	0·878
18	33 52·5	52·5	33 54·5	54·5	33 53·5	0·74	2·72	1·98	1·44	3·40	1·96	53·20	52·95	
19	38 59·5	59·5	39 1·5	1·5	39 0·5	0·75	2·72	1·97	1·45	3·39	1·94	53·25	53·00	0·882
153	15 6 19·5	19·5	15 6 27·5	27·5	15 6 23·5	1·45	2·01	0·56	2·16	2·70	0·54	49·90	49·45	1·775
155	16 35·5	35·5	16 45·5	45·5	16 40·5	1·45	2·00	0·55	2·16	2·69	0·53	49·65	49·25	1·782
159	37 8·5	8·5	37 19·5	19·5	37 14·0	1·47	1·99	0·52	2·17	2·68	0·51	49·40	49·05	1·805
183	17 40 35·5	35·5	17 40 45·5	45·5	17 40 40·5	1·53	1·94	0·41	2·23	2·63	0·40	50·20	50·00	1·952
184	45 42·5	42·5	45 54·5	54·5	45 48·5	1·53	1·94	0·41	2·23	2·62	0·39	50·25	50·10	
185	17 50 50·5	50·5	17 51 2·5	2·5	17 50 56·5	1·53	1·93	0·40	2·24	2·62	0·38	50·35	50·20	1·970

Preliminary corrections.

Correction of the Thermometer-Readings.				Computation of semiarc of vibration $=\alpha$.			
Corrected for index error.			Corrected for effect of exhaustion.	Mean reading of scale $=$ O.	Logarithm of O.	Logarithm of tan α.	True semiarc $=\alpha$.
Upper.	Lower.	Mean.					
°	°	°	°				° ′ ″
53·17	52·755	52·962	53·392	2·250	0·3521825	8·3550433	1 17 51
53·22	52·805	53·012	53·442	2·225	0·3473300	8·3501908	1 16 59
53·32	52·905	53·112	53·542	2·210	0·3443923	8·3472531	1 16 28
53·22	52·655	52·937	53·367	1·970	0·2944662	8·2973270	1 8 10
53·22	52·655	52·937	53·367	1·970	0·2944662	8·2973270	1 8 10
53·27	52·705	52·987	53·417	1·955	0·2911468	8·2940076	1 7 39
49·92	49·21	49·565	49·995	0·550	9·7403627	7·7432235	0 19 2
49·67	49·01	49·340	49·770	0·540	9·7323938	7·7352546	0 18 41
49·43	48·81	49·120	49·550	0·515	9·7118072	7·7146680	0 17 49
50·22	49·74	49·980	50·410	0·505	9·6074550	7·6103158	0 14 1
50·27	49·84	50·055	50·485	0·400	9·6020600	7·6049208	0 13 51
50·37	49·94	50·155	50·585	0·390	9·5910646	7·5939254	0 13 30

Correction for amplitude of arc and rate.

Coincidence.	Vibrations of clock pendulum.	Vibrations of detached pendulum $=n$.	log. $\frac{M \sin(\alpha+\alpha') \sin(\alpha-\alpha')}{32 (\log \sin \alpha - \log \sin \alpha')} = \log A$.	$\log n + \log A$.	Correction for amplitude, +.	Number of infinitely small vibrations during experiment.	Number of infinitely small vibrations per diem.
1 to 2	307·0	305·0	5·5009216	7·9852214	0·0096654		
2 — 3	308·0	306·0	5·4931423	7·9788637	0·0095250		
3 — 17	4294·0	4266·0	5·4412666	9·0712875	0·1178386		
17 — 18 18 — 19	614·0	610·0	5·3871234	8·1724532	0·0148749		
19 — 153	41243·0	40975·0	4·9438104	9·5563294	0·3600223	56271·5256659	86072·700
153 — 155	617·0	613·0	4·2743172	7·0617777	0·0011529		
155 — 159	1233·5	1225·5	4·2457725	7·3340858	0·0021582		
159 — 183	7406·5	7358·5	4·1249578	7·9917471	0·0098118		
183 — 184	308·0	306·0	4·0114300	6·4971514	0·0003142		
184 — 185	308·0	306·0	3·9951628	6·4808842	0·0003026		
	56639·0	56271·0			0·5256659		

Correction for temperature.

Coincidence.	Temperature of pendulum during interval $\frac{t+t'}{2}$.	Number of intervals.	$n\left(\frac{t+t'}{2}\right)$.	Mean temperature of the experiment.	Correction for temperature.	Number of vibrations per diem corrected for temperature.
1 to 2	53°·417	1	53·417			
2 — 3	53·492	1	53·492			
3 — 17	54·454	14	762·356			
17 — 18	53·367	1	53·367			
18 — 19	53·392	1	53·392			
19 — 153	51·706	134	6928·604	51°·649	−0·057 vibration.	86072·643
153 — 155	49·882	2	99·764			
155 — 159	49·660	4	198·640			
159 — 183	49·980	24	1199·520			
183 — 184	50·447	1	50·447			
184 — 185	50·535	1	50·535			
		184	9503·534			

Correction for pressure of air.

Mean pressure in the receiver during the experiment = 1·402 inch = β.

Mean temperature in the receiver during the experiment = 51°·65 = t.

$t-32=19·65$ log = 1·2933626 log β = 0·1467480

log ·0023 = 7·3617278 log C = 9·5105577

·0023 × (t − 32) = 0·045195 log = 8·6550904 9·6573057

log 1 + ·0023(t − 32) = 0·0191973

Correction for pressure = 0·435 log = 9·6381084

Result of the Experiment.

Corrected vibrations: 86072·643

\+ 0·435 for pressure of air.

Number of vibrations *in vacuo*: 86073·078